

# DOCUMENT RESUME

ED 214 947

TM 820 080

AUTHOR Lord, Frederic M.  
 TITLE Standard Error of an Equating by Item Response Theory.  
 INSTITUTION Educational Testing Service, Princeton, N.J.  
 SPONS AGENCY Office of Naval Research, Arlington, Va. Personnel and Training Research Programs Office.  
 REPORT NO ETS-RR-81-49  
 PUB DATE Nov 81  
 CONTRACT N00014-80-C-0402  
 NOTE 35p.  
 EDRS PRICE MF01/PC02 Plus Postage.  
 DESCRIPTORS Comparative Analysis; \*Equated Scores; \*Error of Measurement; \*Latent Trait Theory; \*Mathematical Formulas; Mathematical Models  
 IDENTIFIERS Equipercentile Equating; Linear Equating Method

## ABSTRACT

A formula is derived for the asymptotic standard error of a true-score equating by item response theory (IRT). The equating method is applicable when the two tests to be equated are administered to different groups along with an "anchor test." Numerical standard errors are shown for an actual equating 1) comparing the standard errors of IRT, linear, and equipercentile methods; 2) illustrating the effect of the length of the anchor test on the standard error of the equating. (Author/BW)

\*\*\*\*\*  
 \* Reproductions supplied by EDRS are the best that can be made \*  
 \* from the original document. \*  
 \*\*\*\*\*

STANDARD ERROR OF AN EQUATING BY  
ITEM RESPONSE THEORY

Frederic M. Lord

U.S. DEPARTMENT OF EDUCATION  
NATIONAL INSTITUTE OF EDUCATION  
EDUCATIONAL RESOURCES INFORMATION  
CENTER (ERIC)

☒ This document has been reproduced as  
received from the person or organization  
originating it

☐ Minor changes have been made to improve  
reproduction quality

• Points of view or opinions stated in this docu-  
ment do not necessarily represent official NIE  
position or policy

PERMISSION TO REPRODUCE THIS  
MATERIAL HAS BEEN GRANTED BY

the Office of

Naval Research

TO THE EDUCATIONAL RESOURCES  
INFORMATION CENTER (ERIC) "

This research was sponsored in part by the  
Personnel and Training Research Programs  
Psychological Sciences Division  
Office of Naval Research, under  
Contract No. N00014-80-C-0402

Contract Authority Identification Number  
NR No. 150-453

Frederic M. Lord, Principal Investigator



Educational Testing Service  
Princeton, New Jersey

November 1981

Reproduction in whole or in part is permitted  
for any purpose of the United States Government.

Approved for public release; distribution  
unlimited.

STANDARD ERROR OF AN EQUATING BY  
ITEM RESPONSE THEORY

Frederic M. Lord

This research was sponsored in part by the  
Personnel and Training Research Programs  
Psychological Sciences Division  
Office of Naval Research, under  
Contract No. N00014-80-C-0402

Contract Authority Identification Number  
NR No. 150-453

Frederic M. Lord, Principal Investigator

Educational Testing Service

Princeton, New Jersey

November 1981

Reproduction in whole or in part is permitted  
for any purpose of the United States Government.

Approved for public release; distribution  
unlimited.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1 REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4 TITLE (and Subtitle) Standard Error of an Equating by Item Response Theory.		5 TYPE OF REPORT & PERIOD COVERED Technical Report
		6 PERFORMING ORG. REPORT NUMBER Research Report 81-49
7. AUTHOR(s) Frederic M. Lord		8. CONTRACT OR GRANT NUMBER(s) N00014-80-C-0402
9 PERFORMING ORGANIZATION NAME AND ADDRESS Educational Testing Service Princeton, NJ 08541		10 PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 150-453
11 CONTROLLING OFFICE NAME AND ADDRESS Personnel and Training Research Programs Office of Naval Research (Code*458)		12 REPORT DATE November 1981
		13 NUMBER OF PAGES 18
14 MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15 SECURITY CLASS (of this report)
		15a DECLASSIFICATION/DOWNGRADING SCHEDULE
16 DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17 DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18 SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Equating, Item Response Theory, Standard Error, Mental Tests		
20 ABSTRACT (Continue on reverse side if necessary and identify by block number) A formula is derived for the asymptotic standard error of a true-score equating by item response theory. The equating method is applicable when the two tests to be equated are administered to different groups along with an 'anchor test.' Numerical standard errors are shown for an actual equating 1) comparing the standard errors of IRT, linear, and equipercentile methods; 2) illustrating the effect of the length of the anchor test on the standard error of the equating.		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE

S N 0102-LF-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

## Standard Error of an Equating by Item Response Theory

### Abstract

A formula is derived for the asymptotic standard error of a true-score equating by item response theory. The equating method is applicable when the two tests to be equated are administered to different groups along with an 'anchor test.' Numerical standard errors are shown for an actual equating 1) comparing the standard errors of IRT, linear, and equipercentile methods; 2) illustrating the effect of the length of the anchor test on the standard error of the equating.

### Standard Error of an Equating by Item Response Theory\*

In item response theory (IRT), an examinee's expected number-right score,  $\xi$  on test  $X$  is equal to the test characteristic function evaluated at the examinee's ability level  $\theta$  :

$$\xi = \sum_{g=1}^{n_x} p_g(\theta) \quad (1')$$

where  $p_i(\theta)$  is the item response function, the probability of a correct answer to item  $i$  at ability level  $\theta$  . If we have a second test,  $Y$  , measuring the same ability as  $X$  , the expected number-right score  $\eta$  on this test may be written as

$$\eta = \sum_{h=1}^{n_y} p_h(\theta) \quad (4')$$

Equations (1') and (4') are parametric equations for the functional relationship between  $\xi$  and  $\eta$  . Note that this relationship is an exact mathematical one, not a statistical association. Given any  $\theta$  , (1') and (4') determine a pair of values,  $\xi$  and  $\eta$  , that represent the same ability level as  $\theta$  . Pairs of values  $(\xi, \eta)$  determined in this way are equated. In practice, it is often assumed that the functional relationship of  $\eta$  to  $\xi$  given by (1') and (4') can also be applied to actual number-right scores on the two tests, producing an equating of these scores.

---

\*This work was supported in part by contract N00014-80-C-0402, project designation NR 150-453 between the Office of Naval Research and Educational Testing Service. Reproduction in whole or in part is permitted for any purpose of the United States Government.

Here, we simply deal with the sampling errors in estimating the equating relationship of  $\eta$  to  $\xi$ . In (1') and (4'), estimated item parameters must be used. These are the source of the sampling errors in IRT equating. Note that the ability estimates for individual examinees are not used in (1') and (4') and thus will not appear in our formulas. Until now, the sampling errors of IRT equatings have never been estimated.

#### Data

In IRT equating, we frequently have a set of common items that are administered to all examinees. These are needed in order to get Test Y item parameters on the same scale as Test X item parameters. If the common items are external to tests X and Y, as assumed here, the common items are called the anchor test, or, in the present report, Test W. The sampling variance formulas to be obtained here can be modified in obvious ways for the case where some or all of the common items are internal to the tests that are being equated.

Designate the examinees who took both Tests X and W as Group 1; designate the examinees who took Tests Y and W as Group 2. Typically, every examinee falls in one of these two groups.

In practice when there is a series of test forms A, B, ..., X, Y, Z, ... (say), the 'Group 1' data on Test X are processed as soon as they become available in order to equate Test X to the preceding form. When the Group 2 data become available at some later date, it is often considered uneconomical to rerun the Group 1 data, so Group 2 is

run by itself. This case, where item parameters for Groups 1 and 2 are estimated separately, is the case to be considered here. (The simplifying assumption that is used below to approximate the sampling variances of the estimated item parameters is not available in the alternative case where Groups 1 and 2 are pooled and all parameters, estimated simultaneously.)

#### New Equating Formulas

When parameters are estimated separately for groups 1 and 2, the item parameters and  $\theta$  in (4') have a different origin and scale from the item parameters and  $\theta$  in (1'). It is thus no longer possible simply to eliminate  $\theta$  from (1') and (4') to obtain the relation of  $\eta$  to  $\xi$ . The customary procedure in this situation is to use the anchor test to transform the Group 2 item parameters on to the scale of the Group 1 item parameters. This procedure adds to the sampling variance of the transformed item parameters and greatly complicates any determination of the sampling variance of the subsequent equating. The procedures and formulas given below avoid this problem since they avoid any transformation of item parameters.

Equations (1') and (4') remain unchanged except that additional subscripts (explained below) are used. In particular, the symbols  $\theta_1$  and  $\theta_2$  must be distinguished because groups 1 and 2 use different ability scales:

$$\xi = \sum_g P_{g1}(\theta_1) \quad (1)$$



$$\eta = \sum_g P_{g4}(\theta_2) \quad (4)$$

The item response functions here are written  $P_{gp}$  where  $p = 1, 2, 3, 4$  refers to (test X, group 1), (test W, group 1), (test W, group 2), and (test Y, group 2) respectively, and  $g = 1, 2, \dots, n_p$  where  $n_p$  is the number of items in the appropriate test.

Let us write down similar equations for the expected number-right score  $\omega$  on anchor test W:

$$\omega = \sum_g P_{g2}(\theta_1) \quad (2)$$

$$\omega = \sum_g P_{g3}(\theta_2) \quad (3)$$

The equation numbering keeps the tests in convenient order. The desired equation relation between  $\eta$  and  $\xi$  can be obtained by eliminating  $\theta_1$ ,  $\theta_2$ , and  $\omega$  from these four equations.

Computer programs are available for equating  $\eta$  to  $\xi$  by eliminating  $\theta$  from (1') and (4'). These same programs can be used to equate  $\omega$  to  $\xi$  in one step, using (1) and (2), then to equate  $\eta$  to  $\omega$  in a second step using (3) and (4). This produces an equating of  $\eta$  to  $\xi$  for the presently relevant situation where Group 1 and Group 2 parameters are not on the same scale.

An estimated equating is obtained from (1) -- (4) after replacing the true item parameters by their maximum likelihood estimates. Using carets to denote this change, we have

$$\xi = \sum_g \hat{p}_{g1}(\theta_1) \quad , \quad (1'')$$

$$\hat{\omega} = \sum_g \hat{p}_{g2}(\theta_1) \quad , \quad (2'')$$

$$\hat{\omega} = \sum_g \hat{p}_{g3}(\theta_2) \quad , \quad (3'')$$

$$\hat{\eta} = \sum_g \hat{p}_{g4}(\theta_2) \quad . \quad (4'')$$

These equations show that  $\hat{\eta}$  is a function of all the estimated item parameters together with the specified value of  $\xi$ .

#### Derivatives

For item  $g$ , instead of using  $a_g$ ,  $b_g$ , and  $c_g$  to denote the three parameters commonly used in IRT, let us use  $t_{1gp}$ ,  $t_{2gp}$ , and  $t_{3gp}$ , respectively. We will need certain derivatives for  $r = 1, 2, 3$ , obtained from (1'')-(4''):

$$\frac{\partial \hat{\eta}}{\partial t_{rg4}} = p_{g4}^{(r)}(\theta_2) \quad , \quad (5)$$

$$\frac{\partial \hat{\omega}}{\partial t_{rg3}} = p_{g3}^{(r)}(\theta_2) \quad ,$$

$$\frac{\partial \hat{\omega}}{\partial t_{rg2}} = p_{g2}^{(r)}(\theta_1) \quad ,$$

where  $p_{gp}^{(r)}$  denotes the derivative of  $p_{gp}$  with respect to  $t_{rgp}$ .

Similarly,

$$\frac{\partial \eta}{\partial \theta_2} = \sum_g P'_{g4}(\theta_2) ,$$

$$\frac{\partial \omega}{\partial \theta_1} = \sum_g P'_{g2}(\theta_1)$$

where  $P'$  denotes a derivative with respect to  $\theta$ . Using the formula

for the derivative of an implicit function, we also find from (1'')-(4'')

for  $r = 1, 2, 3$

$$\frac{\partial \theta_2}{\partial t_{rg3}} = - \frac{P_{g3}^{(r)}(\theta_2)}{\sum_g P'_{g3}(\theta_2)} ,$$

$$\frac{\partial \theta_1}{\partial t_{rg1}} = - \frac{P_{g1}^{(r)}(\theta_1)}{\sum_g P'_{g1}(\theta_1)} ,$$

$$\frac{\partial \theta_2}{\partial \omega} = \frac{1}{\sum_g P'_{g3}(\theta_2)}$$

Using the chain rule for derivatives, we find from the above

formulas:

$$\frac{\partial \eta}{\partial t_{rg3}} = \frac{\partial \eta}{\partial \theta_2} \frac{\partial \theta_2}{\partial t_{rg3}} = -P_{g3}^{(r)}(\theta_2) \frac{\sum_g P'_{g4}(\theta_2)}{\sum_g P'_{g3}(\theta_2)} , \quad (6)$$

$$\frac{\partial \eta}{\partial t_{rg2}} = \frac{\partial \eta}{\partial \theta_2} \frac{\partial \theta_2}{\partial \omega} \frac{\partial \omega}{\partial t_{rg2}} = p_{g2}^{(r)}(\theta_1) \frac{\sum_g P'_{g4}(\theta_2)}{\sum_g P'_{g3}(\theta_2)} \quad (7)$$

$$\frac{\partial \eta}{\partial t_{rg1}} = \frac{\partial \eta}{\partial \theta_2} \frac{\partial \theta_2}{\partial \omega} \frac{\partial \omega}{\partial \theta_1} \frac{\partial \theta_1}{\partial t_{rg1}} = -p_{g1}^{(r)}(\theta_1) \frac{\sum_g P'_{g2}(\theta_1)}{\sum_g P'_{g1}(\theta_1)} \frac{\sum_g P'_{g4}(\theta_2)}{\sum_g P'_{g3}(\theta_2)} \quad (8)$$

Given  $\xi$ , we are now in a position to express  $\hat{\eta}$  as a series in powers of  $\hat{t}_{rgp} - t_{rgp}$  ( $r = 1, 2, 3$ ;  $g = 1, 2, \dots, n_p$ ;  $p = 1, 2, 3, 4$ ).

We will write  $\eta'_{rgp}$  instead of  $\partial \eta / \partial t_{rgp}$  and  $\eta''_{rgpshq}$  instead of  $\partial^2 \eta / \partial t_{rgp} \partial t_{shq}$ .

$$\begin{aligned} \hat{\eta} = & \eta + \sum_p \sum_g \sum_r (\hat{t}_{rgp} - t_{rgp}) \eta'_{rgp} \\ & + \frac{1}{2} \sum_p \sum_q \sum_g \sum_h \sum_r \sum_s (\hat{t}_{rgp} - t_{rgp})(\hat{t}_{shq} - t_{shq}) \eta''_{rgpshq} + \dots \quad (9) \end{aligned}$$

### Sampling Variance

Transposing, squaring, and taking expectations, we find from (9) for fixed  $\xi$ ,

$$\text{Var } \hat{\eta} = \delta(\hat{\eta} - \eta)^2 = \sum_p \sum_q \sum_g \sum_h \sum_r \sum_s \eta'_{rgp} \eta'_{shq} \text{Cov}(\hat{t}_{rgp}, \hat{t}_{shq}) + \dots$$

When item parameters and abilities are both estimated simultaneously by maximum likelihood, it is not practical to use the usual sampling covariance formulas for all estimators simultaneously. As a rough approximation, it is customary (Lord, 1980, Section 12.3) to use instead the (simpler) formulas for the case where the ability parameters are known. We will use this rough approximation here to find  $\text{Cov}(\hat{t}_{rgp}, \hat{t}_{shq})$ . Because of this approximation, our sampling variance of equating will be an underestimate.

In this case, all covariances involving two different items are exactly zero, as are all covariances involving a single item administered to two different groups of examinees. All nonzero variances and covariances are inversely proportional to  $N$ , the number of examinees.

We now have

$$\begin{aligned} \text{Var } \hat{\eta} = & \sum_p \sum_g \left[ \sum_{r=1}^3 \sum_{s=1}^3 \{ \eta'_{rgp} \eta'_{sgp} \text{Cov}(\hat{t}_{rgp}, \hat{t}_{sgp}) \} \right. \\ & \left. + \sum_{r=1}^3 \sum_{s=1}^3 \sum_{t=1}^3 \{ \cdot \} + \sum_{r=1}^3 \sum_{s=1}^3 \sum_{t=1}^3 \sum_{u=1}^3 \{ \cdot \} + \dots \right] \end{aligned}$$

Some higher order terms are indicated here in order to make clear that the number of terms under summation signs does not increase too rapidly. The triple summation represents 3 times as many terms as the double summation, but each term in the triple summation is divided by  $N^{3/2}$  whereas each term in the double summation is only divided by  $N$ . When  $N$  is several thousand, it is reasonable to expect that the higher order terms can be neglected, as is customary with asymptotic variances.

Our final asymptotic formula, then is

$$\text{Var } \hat{\eta} = \sum_{p=1}^4 \sum_{g=1}^n \sum_{r=1}^3 \sum_{s=1}^3 \eta'_{rgp} \eta'_{sgp} \text{Cov}(\hat{t}_{rgp}, \hat{t}_{sgp}) \quad (10)$$

The  $\eta'$  values required here are computed from (5) - (8). The covariances are obtained by the usual formulas for covariances of maximum likelihood estimators of item parameters when ability parameters are fixed (Lord, 1980, p. 191).

#### Practical Application

Without data, it is difficult to make inferences about the magnitude of the sampling errors in IRT equating. Will they be larger or smaller than the sampling errors in conventional linear equating? In conventional equipercentile equating? Do sampling errors become large or small at extreme score levels?

Equation (10) has been applied to an equating of the Verbal score on the 90-item Form VSA4 of the Scholastic Aptitude Test (12/73 administration) to the 85-item Form XSA2 Verbal score (4/75 administration). All examinees took an SAT and also a 40-item anchor test. Petersen, Cook, and Stocking (1980) made separate LOGIST runs on the 130 items in the 1973 administration for a sample of 2665 examinees, and on the 125 items in the 1975 administration for a sample of 2686 examinees. They have allowed the use here of their item parameter estimates.

SAT scaled scores are a linear transformation of formula scores (rights minus one-quarter wrongs). Our results here are for the hypothetical case where all examinees answer all items. In this special case formula scores are a linear transformation of number-right scores, so scaled scores are likewise. Since a known linear transformation  $A\xi + B$  of number-right scores  $\xi$  simply multiplies the standard error of  $\eta$  by the constant  $A$ , it is not difficult to obtain scaled-score standard errors from (10). A computer program to do this was written and run by Marilyn Wingersky.

For each of certain specified formula scores on XSA2, Table 1 shows 1) the equivalent scaled score found by the conventional linear procedure usually used for the SAT (Design IV A, Angoff, 1971), 2) the standard error of these equated (scaled) scores as found by the computer program AUTEST (Lord, 1975) assuming the validity of the linear model; also 3) the equivalent scaled score found by the IRT method of this report, and 4) the corresponding scaled-score standard error calculated from (10). The standard errors in Table 1 are best understood in comparison with the standard deviation of scaled scores, which is 106 for XSA2; and in comparison with the classical test theory standard error of measurement (due to imperfect test reliability), which is 31. Clearly the standard error of equating is small compared to the standard error of measurement.

Judging by the IRT standard errors, the equating is definitely nonlinear, at least outside the score range from 350 to 650. The IRT standard errors show a continued sharp increase as the minimum

Table 1

A Comparison of Linear and IRT Equatings and of Their Standard Errors

Selected formula scores*, XSA2	Linear Model		IRT Model	
	Equivalent scaled score	Standard error	Equivalent scaled score	Standard error
84	780	4.6	813.8	2.3
79.74	750	4.2	778.0	4.5
72.70	700	3.6	717.6	4.4
65.65	650	3.1	658.8	3.6
58.61	600	2.5	602.4	2.8
51.57	550	2.1	548.0	2.2
44.52	500	1.7	495.4	2.0
37.48	450	1.5	445.7	2.1
30.43	400	1.6	399.3	2.3
23.39	350	1.8	355.6	2.8
16.35	300	2.3	313.3	3.6
9.30	250	2.8	270.2	4.7
2.26	200	3.3	223.0	7.0
-5	150	3.9	163.5	15.6

\*Although formula score is actually a discrete variable, it is for convenience treated here as continuous.



possible true formula score of -5.5 is approached. At the other end of the score scale, the IRT standard error increases up to a scaled score of 760 and decreases thereafter. The reason for the decrease at the upper end is that for a perfect score, the standard error of this kind of IRT equating is zero. Except at the upper end, the IRT standard error is larger than the linear.

The results of Table 1 are displayed in Figures 1-2. The straight line in Figure 1 shows the linear equating of true formula score on XSA2 to true scaled score on VSA4. The dashed lines are drawn two standard errors above and below the straight line.

Figure 2 similarly displays the curvilinear IRT equating of XSA2 to VSA4 and its standard error. The straight-line extension of the lower end of the equating (middle) line in Figure 2 was obtained by the method described in Lord (1980, pp. 210-211). It is shown in the figure for completeness; but no standard error is shown since there is no good theoretical basis for such an extension.

Table 2 compares present IRT equating with a conventional equipercen- tile equating of XSA2 to VSA4 via the anchor test. In conventional equating, an XSA2 score and a VSA4 score each equipercen- tile equivalent to a given anchor test score are taken to be equivalent to each other. The standard error of the resulting equipercen- tile equating of XSA2 to VSA4 is given by  $\sqrt{SE_{XSA2}^2 + SE_{VSA4}^2}$  where the SE under the radical sign are standard errors of separate equipercen- tile equatings of each test to the anchor test. Formulas for  $SE_{XSA2}$  and  $SE_{VSA4}$  are given in Lord (1981).

Figure 1. Linear equating of true formula score on XSA2 to true scaled score on VSA4. Dashed lines are two scaled-score standard errors above and below equating line.

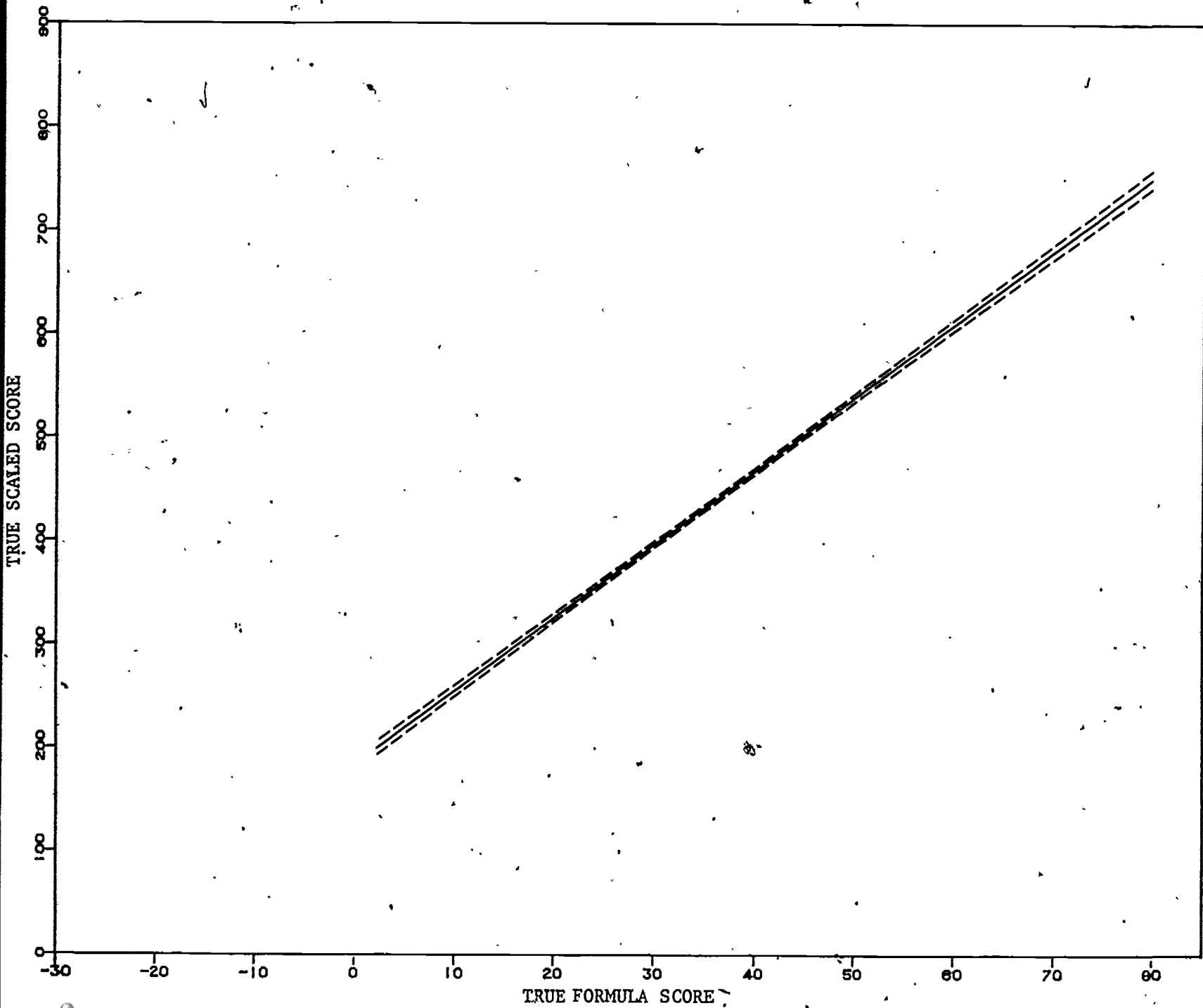


Figure 2. IRT equating of XSA2 formula score to VSA4 scaled score, with two-standard-error bounds.

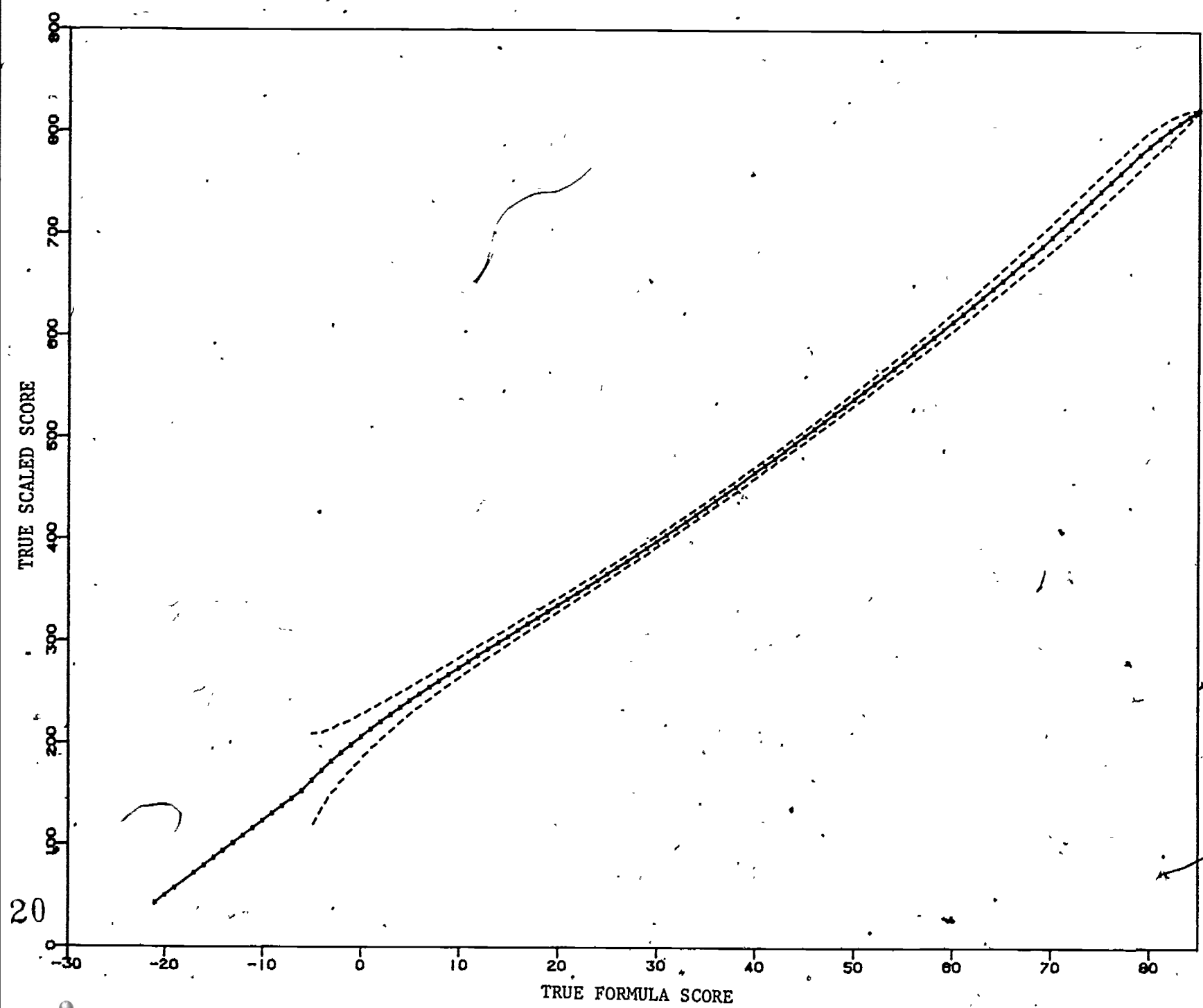


Table 2

A Comparison of Equipercentile and IRT Equating  
and of Their Standard Scores

XSA2 formula score	Equipercentile Method		IRT Model	
	Equivalent scaled score	Standard error	Equivalent scaled score	Standard error
78.1	774	13.47	764	4.68
70.6	722	15.85	700	4.18
64.75	652	10.32	651	3.44
58.9	602	4.97	605	2.78
52.9	558	4.12	558	2.32
47.25	514	3.47	515	2.09
40.1	466	3.44	464	2.05
32.4	417	2.93	412	2.24
25.75	364	3.37	370	2.63
16.1	314	4.07	312	3.62
7.6	242	5.70	259*	5.08
-3.75	195	7.85	175	12.49

Since  $SE_{XSA2}$  and  $SE_{VSA4}$  are estimated from unsmoothed data, the equipercntile standard errors in Table 2 fluctuate somewhat. Nevertheless, it is apparent that the equipercntile method has a much larger standard error above a scaled score of 450. For these data, the IRT method shows a larger standard error than the equipercntile method only when the formula score is negative.

The standard error of equipercntile equating could be reduced by smoothing the frequency distribution of raw scores before equating. Smoothing is undoubtedly desirable as a practical expedient; however, the choice of a smoothing formula is somewhat arbitrary and the smoothing is likely to prevent convergence of the estimated equating to its true value in large samples. Formulas for the standard errors of smoothed equipercntile equating are not presently available.

In order to determine the effect of using a shorter anchor test, every other item in the anchor test was discarded and the data reanalyzed on the basis of the remaining 20-item anchor test. The effect on the standard errors of IRT equating is shown in Table 3. The two equatings agree fairly well. At the point where the equating standard errors are a minimum, halving the length of the anchor test increases the standard error by a factor of about  $\sqrt{2}$ . At the other score points, the effect is less. Given standard errors like those in Table 2, it will now be possible to make a reasonable judgment as to the length necessary for an anchor test.

Table 3

IRT Equatings and Their Scaled-Score Standard Errors,  
a Comparison of Results Using 20- and 40-Item Anchor Tests

XSA2 formula score	Length of Anchor Test			
	20 Items		40 Items	
	Scaled score	Standard error	Scaled score	Standard error
80	787	5.9	780	4.5
70	698	5.3	695	4.1
60	615	3.9	613	2.9
50	540	3.0	536	2.2
40	467	2.7	463	2.0
30	399	3.0	397	2.4
20	336	3.9	335	3.2
10	274	5.4	275	4.6
0	206	9.9	206	8.4

References

- Angoff, W. H. Scales, norms, and equivalent scores. In R. L. Thorndike (Ed.), Educational measurement (2nd ed.). Washington, D.C.: American Council on Education, 1971. Pp. 508-600.
- Lord, F. M. Automated hypothesis tests and standard errors for nonstandard problems. The American Statistician, 1975, 29, 56-59.
- Lord, F. M. Applications of item response theory to practical testing problems. Hillsdale, N.J.: Lawrence Erlbaum Associates, 1980.
- Lord, F. M. The standard error of equipercentile equating. Research Report 81-00. Princeton, N.J.: Educational Testing Service, 1981, in preparation.
- Petersen, N. S., Cook, L. L., & Stocking, M. S. Scale drift: A comparative study of IRT versus linear equating methods. Paper presented at the Fourth International Symposium on Educational Testing, Antwerp, Belgium, 1980.

# DISTRIBUTION LIST

## Navy

- 1 Dr. Ed Aiken  
Navy Personnel R & D Center  
San Diego, CA 92152
- 1 Dr. Jack R. Borsting  
Provost and Academic Dean  
U.S. Naval Postgraduate School  
Monterey, CA 93940
- 1 Dr. Robert Breaux  
Code N-711  
NAVTRAEQUIPCEN  
Orlando, FL 32813
- 1 Chief of Naval Education and  
Training Liason Office  
Air Force Human Resource Laboratory  
Flying Training Division  
Williams Air Force Base, AZ 85224
- 1 CDR Mike Curran  
Office of Naval Research  
800 North Quincy Street  
Code 270  
Arlington, VA 22217
- 1 Dr. Richard Elster  
Department of Administrative Sciences  
Naval Postgraduate School  
Monterey, CA 93940
- 1 Dr. Pat Federico  
Navy Personnel R & D Center  
San Diego, CA 92152
- 1 Mr. Paul Foley  
Navy Personnel R & D Center  
San Diego, CA 92152
- 1 Dr. John Ford  
Navy Personnel R & D Center  
San Diego, CA 92152
- 1 Dr. Patrick R. Harrison  
Psychology Course Director  
Leadership and Law Department (7b)  
Division of Professional Development  
U.S. Naval Academy  
Annapolis, MD 21402
- 1 Dr. Norman J. Kerr  
Chief of Naval Technical Training  
Naval Air Station Memphis (75)  
Millington, TN 38054
- 1 Dr. William L. Maloy  
Principal Civilian Advisor for  
Education and Training  
Naval Training Command, Code 00A  
Pensacola, FL 32508
- 1 Dr. Kneale Marshall  
Scientific Advisor to DCNO(MPT)  
Op01T  
Washington, DC 20370
- 1 Dr. James McBride  
Navy Personnel R & D Center  
San Diego, CA 92152
- 1 Dr. William Montague  
Navy Personnel R & D Center  
San Diego, CA 92152
- 1 Mr. William Nordbrock  
Instructional Program Development  
Building 90  
NET-PDCD  
Great Lakes NTC, IL 60088
- 1 Library, Code P201L  
Navy Personnel R & D Center  
San Diego, CA 92152



- 1 Technical Director  
Navy Personnel R & D  
San Diego, CA 92152
- 6 Commanding Officer  
Naval Research Laboratory  
Code 2627  
Washington, DC 20390
- 1 Psychologist  
ONR Branch Office  
Building 114, Section D  
666 Summer Street  
Boston, MA 02210
- 1 Office of Naval Research  
Code 437  
800 North Quincy Street  
Arlington, VA 22217
- 5 Personnel and Training Research  
Programs  
Code 458  
Office of Naval Research  
Arlington, VA 22217
- 1 Psychologist  
ONR Branch Office  
1030 East Green Street  
Pasadena, CA 91101
- 1 Office of the Chief of Naval Operations  
Research Development and Studies Branch  
OP-115  
Washington, DC 20350
- 1 LT Frank C. Petho, MSC, USN (Ph.D.)  
Selection and Training Research Division  
Human Performance Sciences Department  
Naval Aerospace Medical Research Lab.  
Pensacola, FL 32508
- 1 Dr. Bernard Rimland (O3B)  
Navy Personnel R & D Center  
San Diego, CA 92152
- 1 Mr. Arnold Rubenstein  
Office of Naval Technology  
800 N. Quincy Street  
Arlington, VA 22217
- 1 Dr. Worth Scanland, Director  
Research, Development, Test  
and Evaluation  
N-5  
Naval Education and Training Command  
NAS  
Pensacola, FL 32508
- 1 Dr. Robert G. Smith  
Office of Chief of Naval Operations  
OP-987H  
Washington, DC 20350
- 1 Dr. Alfred F. Smode  
Training Analysis and Evaluation Group  
Department of the Navy  
Orlando, FL 32813
- 1 Dr. Richard Sorensen  
Navy Personnel R & D Center  
San Diego, CA 92152
- 1 Mr. J. B. Sympton  
Naval Personnel R & D Center  
San Diego, CA 92152
- 1 Dr. Ronald Weitzman  
Code 54 WZ  
Department of Administrative Services  
U.S. Naval Postgraduate School  
Monterey, CA 93940
- 1 Dr. Robert Wherry  
562 Mallard Drive  
Chalfont, PA 18914

- 1 Dr. Robert Wisher  
Code 309  
Navy Personnel R & D Center  
San Diego, CA 92152
- 1 Dr. Martin F. Wiskoff  
Navy Personnel R & D Center  
San Diego, CA 92152
- 1 Mr. Ted M. I. Yellen  
Technical Information Office  
Code 201  
Navy Personnel R & D Center  
San Diego, CA 92152

Army

- 1 Technical Director  
U.S. Army Research Institute for the  
Behavioral and Social Sciences  
5001 Eisenhower Avenue  
Alexandria, VA 22333
- 1 Dr. Myron Fischl  
U.S. Army Research Institute for the  
Social and Behavioral Sciences  
5001 Eisenhower Avenue  
Alexandria, VA 22333
- 1 Dr. Dexter Fletcher  
U.S. Army Research Institute  
5001 Eisenhower Avenue  
Alexandria, VA 22333
- 1 COL Frank Hart  
Army Research Institute for the  
Behavioral & Social Sciences  
5001 Eisenhower Blvd.  
Alexandria, VA 22333
- 1 Dr. Michael Kaplan  
U.S. Army Research Institute  
5001 Eisenhower Avenue  
Alexandria, VA 22333

- 1 Dr. Milton S. Katz  
Training Technical Area  
U.S. Army Research Institute  
5001 Eisenhower Avenue  
Alexandria, VA 22333
- 1 Dr. Harold F. O'Neil, Jr.  
Attn: PERI-OK  
Army Research Institute  
5001 Eisenhower Avenue  
Alexandria, VA 22333
- 1 Dr. Robert Sasmor  
U.S. Army Research Institute for  
the Social and Behavioral Sciences  
5001 Eisenhower Avenue  
Alexandria, VA 22333
- 1 Commandant  
U.S. Army Institute of Administration  
Attn: Dr. Sherrill  
Ft. Benjamin Harrison, IN 46256
- 1 Dr. Frederick Steinheiser  
Department of the Navy  
Chief of Naval Operations  
OP-113  
Washington, DC 20350
- 1 Dr. Joseph Ward  
U.S. Army Research Institute  
5001 Eisenhower Avenue  
Alexandria, VA 22333

Air Force

- 1 Air Force Human Resources Laboratory  
AFHRL/MPD  
Brooks Air Force Base, TX 78235
- 1 U.S. Air Force Office of  
Scientific Research  
Life Sciences Directorate  
Bolling Air Force Base  
Washington, DC 20332

1 Air University Library  
AUL/LSE 76/443  
Maxwell Air Force Base, AL 36112

1 Dr. Earl A. Alluisti  
HQ, AFHRL (AFSC)  
Brooks Air Force Base, TX 78235

1 Mr. Raymond E. Christal  
AFHRL/MO  
Brooks Air Force Base, TX 78235

1 Dr. Genevieve Haddad  
Program Manager  
Life Sciences Directorate  
AFOSR  
Bolling Air Force Base  
Washington, DC 20332

1 Dr. Ross L. Morgan  
AFHRL/LR  
Wright-Patterson AFB, OH 45433

1 Research and Measurement Division  
Research Branch, AFMPC/MPCYPR  
Randolph Air Force Base, TX 78148

1 Dr. Malcolm Ree  
AFHRL/MP  
Brooks Air Force Base, TX 78235

1 Dr. Marty Rockway  
Technical Director  
AFHRL(OT)  
Williams Air Force Base, AZ 58224

#### Marines

1 Dr. H. William Greenup  
Education Advisor (EO31)  
Education Center, MCDEC  
Quantico, VA 22134

1 Director, Office of Manpower  
Utilization  
HQ, Marine Corps (MPU)  
ECB, Building 2009  
Quantico, VA 22134

1 MAJ Michael L. Patrow, USMC  
Headquarters, Marine Corps  
Code MPI-20  
Washington, DC 20380

1 Dr. A. L. Slafkosky  
Scientific Advisor  
Code RD-1  
HQ, U.S. Marine Corps  
Washington, DC 20380

#### Coast Guard

1 Mr. Thomas A. Warm  
U.S. Coast Guard Institute  
P.O. Substation 18  
Oklahoma City, OK 73169

#### Other DoD

1 DARPA  
1400 Wilson Boulevard  
Arlington, VA 22209

12 Defense Technical Information Center  
Cameron Station, Building 5  
Attn: TC  
Alexandria, VA 22314

1 Dr. William Graham  
Testing Directorate  
MEPCOM/MEPCT-P  
Ft. Sheridan, IL 60037

1 Military Assistant for Training  
and Personnel Technology  
Office of the Under Secretary of  
Defense for Research and Engineering  
Room 3D129, The Pentagon  
Washington, DC 20301

1 Dr. Wayne Sellman  
Office of the Assistant Secretary  
of Defense (MRAL)  
2B269 The Pentagon  
Washington, DC 20301

Civil Government

1 Dr. Susan Chipman  
Learning and Development  
National Institute of Education  
1200 19th Street, NW  
Washington, DC 20208

1 Mr. Richard McKillip  
Personnel R & D Center  
Office of Personnel Management  
1900 E Street, NW  
Washington, DC 20415

1 Dr. Arthur Haimel  
National Institute of Education  
1200 19th Street, N.W.  
Washington, DC 20208

1 Dr. Andrew R. Molnar  
Science Education Development  
and Research  
National Science Foundation  
Washington, DC 20550

1 Dr. Vern W. Urry  
Personnel R & D Center  
Office of Personnel Management  
1900 E Street, NW  
Washington, DC 20415

1 Dr. Joseph L. Young, Director  
Memory and Cognitive Processes  
National Science Foundation  
Washington, DC 20550

Non-Government

1 Dr. Erling B. Andersen  
Department of Statistics  
Studiestraede 6  
1455 Copenhagen  
DENMARK

1 Psychological Research Unit  
Department of Defense (Army Office)  
Campbell Park Offices  
Canberra, ACT 2600  
AUSTRALIA

1 Dr. Alan Baddeley  
Medical research Council  
Applied Psychology Unit  
15' Chaucer Road  
Cambridge CB2 2EF  
ENGLAND

1 Dr. Isaac Bejar  
Educational Testing Service  
Princeton, NJ 08541

1 Dr. Menucha Birenbaum  
School of Education  
Tel Aviv University  
Tel Aviv, Ramat Aviv 69978  
ISRAEL

1 Dr. Werner Birke  
DezWPs im Streitkrafteamt  
Postfach 20 50 3  
D-5300 Bonn 2  
WEST GERMANY

- 1 Dr. R. Darrell Bock  
Department of Education  
University of Chicago  
Chicago, IL 60637
- 1 Liaison Scientists  
Office of Naval Research  
Branch Office, London  
Box 39  
FPO, NY 09510
- 1 Dr. Robert Brennan  
American College Testing Programs  
P.O. Box 168  
Iowa City, IA 52240
- 1 Dr. John B. Carroll  
Psychometric Laboratory  
University of North Carolina  
Davie Hall 013A  
Chapel Hill, NC 27514
- 1 Charles Myers Library  
Livingstone House  
Livingstone Road  
Stratford  
London E15 2LJ  
ENGLAND
- 1 Dr. Kenneth E. Clark  
College of Arts and Sciences  
University of Rochester  
River Campus Station  
Rochester, NY 14627
- 1 Dr. Norman Cliff  
Department of Psychology  
University of Southern California  
University Park  
Los Angeles, CA 90007
- 1 Dr. William E. Coffman  
Director, Iowa Testing Programs  
334 Lindquist Center  
University of Iowa  
Iowa City, IA 52242
- 1 Dr. Allan M. Collins  
Bolt, Beranek, & Newman, Inc.  
50 Moulton Street  
Cambridge, MA 02138
- 1 Dr. Meredith P. Crawford  
American Psychological Association  
1200 17th Street, N  
Washington, DC 20036
- 1 Dr. Hans Crombag  
Education Research Center  
University of Leyden  
Boerhaavelaan 2  
2334 EN Leyden  
THE NETHERLANDS
- 1 Dr. Fritz Drasgow  
Yale School of Organization and  
Management  
Yale University  
Box 1A  
New Haven, CT 06520
- 1 LCOL J. C. Eggenberger  
Directorate of Personnel  
Applied Research  
National Defence Hq.  
101 Colonel By Drive  
Ottawa, KIA UK2  
CANADA
- 1 Dr. Benjamin A. Fairbank, Jr.  
McFann-Gray and Associates, Inc.  
5825 Callaghan  
Suite 225  
San Antonio, TX 78228

- 1 Dr. Leonard Feldt  
Lindquist Center for Measurement  
University of Iowa  
Iowa City, IA 52242
- 1 Dr. Richard L. Ferguson  
The American College Testing Program  
P.O. Box 168  
Iowa City, IA 52240
- 1 Dr. Victor Fields  
Department of Psychology  
Montgomery College  
Rockville, MD 20850
- 1 Univ. Prof. Dr. Gerhard Fischer  
Psychologisches Institut der  
Universitat Wien  
Liebiggasse 5/3  
A 1010 Wien  
AUSTRIA
- 1 Prof. Donald Fitzgerald  
University of New England  
Armidale, New South Wales 2351  
AUSTRALIA
- 1 Dr. Edwin A. Fleishman  
Advanced Research Resources Organization  
Suite 900  
4330 East West Highway  
Washington, DC 20014
- 1 Dr. John R. Frederiksen  
Bolt, Beranek, and Newman  
50 Moulton Street  
Cambridge, MA 02138
- 1 Dr. Robert Glaser  
LRDC  
University of Pittsburgh  
3939 O'Hara Street  
Pittsburgh, PA 15213
- 1 Dr. Bert Green  
Department of Psychology  
Johns Hopkins University  
Charles and 34th Streets  
Baltimore, MD 21218
- 1 Dr. Ron Hambleton  
School of Education  
University of Massachusetts  
Amherst, MA 01002
- 1 Dr. Lloyd Humphreys  
Department of Psychology  
University of Illinois  
Champaign, IL 61820
- 1 Library  
HumRRO/Western Division  
27857 Berwick Drive  
Carmel, CA 93921
- 1 Dr. Steven Hunka  
Department of Education  
University of Alberta  
Edmonton, Alberta  
CANADA
- 1 Dr. Earl Hunt  
Department of Psychology  
University of Washington  
Seattle, WA 98105
- 1 Dr. Jack Hunter  
2122 Coolidge Street  
Lansing, MI 48906
- 1 Dr. Huynh Huynh  
College of Education  
University of South Carolina  
Columbia, SC 29208
- 1 Mr. Marlin Kroger  
1117 Via Goleta  
Palos Verdes Estates, CA 90274

- 1 Dr. Michael Levine  
Department of Educational Psychology  
210 Education Building  
University of Illinois  
Champaign, IL 61801
- 1 Dr. Charles Lewis  
Faculteit Sociale Wetenschappen  
Rijksuniversiteit Groningen  
Oude Boteringestraat 23  
9712GC Groningen  
NETHERLANDS
- 1 Dr. Robert Linn  
College of Education  
University of Illinois  
Urbana, IL 61801
- 1 Dr. James Lumsden  
Department of Psychology  
University of Western Australia  
Nedlands, Western Australia 6009  
AUSTRALIA
- 1 Dr. Gary Marco  
Educational Testing Service  
Princeton, NJ 08541
- 1 Dr. Scott Maxwell  
Department of Psychology  
University of Houston  
Houston, TX 77004
- 1 Dr. Samuel T. Mayo  
Loyola University of Chicago  
820 North Michigan Avenue  
Chicago, IL 60611
- 1 Dr. Allen Munro  
Behavioral Technology Laboratories  
1845 Elena Avenue  
Fourth Floor  
Redondo Beach, CA 90277
- 1 Dr. Melvin R. Novick  
356 Lindquist Center for Measurement  
University of Iowa  
Iowa City, IA 52242
- 1 Dr. Jesse Orlansky  
Institute for Defense Analyses  
400 Army Navy Drive  
Arlington, VA 22202
- 1 Dr. Wayne M. Patience  
American Council on Education  
GED Testing Service, Suite 20  
One Dupont Circle, NW  
Washington, DC 20036
- 1 Dr. James A. Paulson  
Portland State University  
P.O. Box 751  
Portland, OR 97207
- 1 Mr. Luigi Petrullo  
2431 North Edgewood Street  
Arlington, VA 22207
- 1 Dr. Diane H. Ramsey-Klee  
R-K Research and System Design  
3947 Ridgmont Drive  
Malibu, CA 90265
- 1 Mr. Minrat M. L. Rauch  
P II 4  
Bundesministerium der Verteidigung  
Postfach 1328  
D-53 Bonn 1  
GERMANY
- 1 Dr. Mark D. Reckase  
Educational Psychology Department  
University of Missouri-Columbia  
4 Hill Hall  
Columbia, MO 65211

- 1 Dr. Leonard L. Rosenbaum, Chairman  
Department of Psychology  
Montgomery College  
Rockville, MD 20850
- 1 Dr. Ernst Z. Rothkopf  
Bell Laboratories  
600 Mountain Avenue  
Murray Hill, NJ 07974
- 1 Dr. Lawrence Rudner  
403 Elm Avenue  
Takoma Park, MD 20012
- 1 Dr. J. Ryan  
Department of Education  
University of South Carolina  
Columbia, SC 29208
- 1 Prof. Fumiko Samejima  
Department of Psychology  
University of Tennessee  
Knoxville, TN 37916
- 1 Dr. Frank L. Schmidt  
Department of Psychology  
Building GG  
George Washington University  
Washington, DC 20052
- 1 Dr. Robert J. Seidel  
Instructional Technology Group  
HumRRO  
300 North Washington Street  
Alexandria, VA 22314
- 1 Committee on Cognitive Research  
c/o Dr. Lonnie K. Sherrod  
Social Science Research Council  
605 Third Avenue  
New York, NY 10016
- 1 Dr. Kazuo Shigemasu  
University of Tohoku  
Department of Educational Psychology  
Kawauchi, Sendai 980  
JAPAN
- 1 Dr. Edwin Shirkey  
Department of Psychology  
University of Central Florida  
Orlando, FL 32816
- 1 Dr. Richard Snow  
School of Education  
Stanford University  
Stanford, CA 94305
- 1 Dr. Robert Sternberg  
Department of Psychology  
Yale University  
Box 11A, Yale Station  
New Haven, CT 06520
- 1 Dr. Albert Stevens  
Bolt, Beranek, and Newman, Inc.  
50 Moulton Street  
Cambridge, MA 02138
- 1 Dr. Hariharan Swaminathan  
Laboratory of Psychometric and  
Evaluation Research  
School of Education  
University of Massachusetts  
Amherst, MA 01003
- 1 Dr. Kikumi Tatsuoka  
Computer Based Education Research  
Laboratory  
252 Engineering Research Laboratory  
University of Illinois  
Urbana, IL 61801
- 1 Dr. David Thissen  
Department of Psychology  
University of Kansas  
Lawrence, KS 66044



- 1 Dr. Robert Tsutakawa  
Department of Statistics  
University of Missouri  
Columbia, MO 65201
- 1 Dr. David Vale  
Assessment Systems  
Corporation  
2395 University Avenue  
Suite 306  
St. Paul, MN 55114
- 1 Dr. Howard Wainer  
Educational Testing Service  
Princeton, NJ 08541
- 1 Dr. Thomas Wallsten  
Psychometric Laboratory  
Davie Hall 013A  
University of North Carolina  
Chapel Hill, NC 27514
- 1 Dr. Phyllis Weaver  
Graduate School of Education  
Harvard University  
200 Larsen Hall, Appian Way  
Cambridge, MA 02138
- 1 Dr. David J. Weiss  
Nob0 Elliott Hall  
University of Minnesota  
75 East River Road  
Minneapolis, MN 55455
- 1 Dr. Susan E. Whitely  
Psychology Department  
University of Kansas  
Lawrence, KS 66044
- 1 Dr. Wolfgang Wildgrube  
Streitkraefteamt  
Box 20 50 03  
D-5300 Bonn 2  
WEST GERMANY